Simulating Photon Motion near Black Holes

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Calculating paths of photons along space-time curvature is of particular interest in understanding physical observations and formulating and testing theories of general relativity. Typical continuous-time solutions of photon motion make simulations in complex situations prohibitively difficult. This paper demonstrates how Fermat's Surface approximation of curvature as a continuous refractive index can be used to calculate geodesic paths of photons on a discrete-time, step-by-step basis in an interactive and real-time simulation. Doing so allows the estimation of photon motion in complex setups such as areas where multiple, in-motion black holes are present. Despite some strange effects in edge cases, this simulation proves to be a useful tool for demonstrating different facets of general relativity and curvature.

I. INTRODUCTION

In a gravitational field that distorts space-time, passing photons will follow a geodesic path along that curvature. While any massive or energetic object will curve space-time, black holes are of particular interest in studying curvature due to their relative strength and small size. Gravitational lensing is an important topic in the study of astrophysics and general relativity. Gravitational lensing describes an optical effect that can magnify or distort an image when strong curvature is present between the image source and the observer. More generally, this refers to the non-linear path of light rays in space as they follow a geodesic along the space-time curvature. Computational simulation of this unusual photon behavior is an important tool for studying and explaining real-world observations of space and the origins of the universe.

Typical computer models of gravitational lensing simulate the distortion of an image that an observer might see from the other side of a massive object (Figure 1). However, there are relatively few simulations of this phenomenon from the perspective of an outside observer. Such a simulation could give greater insight into the motions of individual photons, specifically in regards to time dilation, redshift, and orbits. The time delay of certain portions of the image may also be better visualized using this approach.

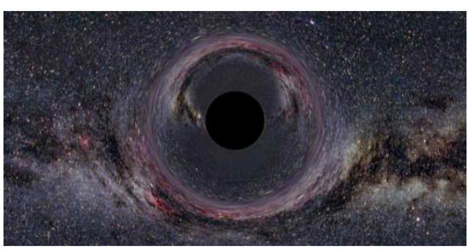

Figure 1 – A computer simulation of gravitational lensing effects on an image of the Milky Way (Wikipedia).

The path of photons following a curved space-time is mathematically described by a Schwarzschild geodesic[1] equation. Solving for a geodesic path from this approach, however, is difficult for humans and computers alike. Instead, there are a number of solutions and approximations[2] to the problem in the specific case of photon motion that simplify the task. Some of these methods include the Born Approximation, the Thin Lens Approximation, Deflection Potential, and the Fermat Surface.

While many simulations opt for finding continuous solutions to photon paths, I decided to take a discrete-time approach, in which a new linear trajectory is calculated at every finite time step in the simulation. This should allow for real-time simulations in more complex situations, for example in systems with multiple, moving black holes, that would otherwise be prohibitively difficult in a continuous solution. For this paper, I have chosen to use the Fermat Surface approach to solving photon paths.

II. PHOTON MOTION

After a lengthy derivation solving the geodesic for a photon at zero eigentime, the Fermat Surface approach approximates the path of light as moving through a medium with a continuously changing refractive index[7]. This refractive index for a given point is calculated using Equation 1[3] where Φ is the gravitational potential, given as a Newtonian solution with Schwarzschild correction in Equation 2[1][8]. G is the Gravitational Constant, M is the mass of the black hole, and α is the angular velocity of the photon with respect to the black hole. For multiple black holes, Φ is simply the sum of all potential.

$$\mathrm{n} = \frac{\mathrm{c}}{\mathrm{c}'} = \frac{1}{1+\frac{2\Phi}{c^2}} \tag{1}$$

$$\Phi = -\frac{GM}{r} - \frac{GMr\alpha^2}{c^2} \tag{2}$$

Treating space-time curvature as a changing index of refraction *n* allows for relatively simple calculation of time dilation (the $\frac{c}{c'}$ term), and of photon trajectory (Figure 2) at a given interface, as given by Snell's Law in Equation 3.

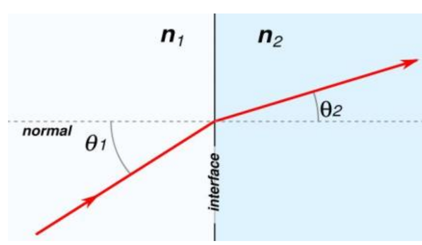

Figure 2 – A demonstration of the path of light upon entering a medium with higher index of refraction (http://physics.technion.ac.il/).

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{3}$$

III. DISCRETE-TIME SOLUTION

The Fermat Surface approach to photon motion can be solved continuously using Fermat's Principle of least action – this involves solving for the equation of motion in an Euler equation. This is not an easy task, and it becomes nigh-impossible to do for spaces with multiple massive bodies. As previously stated, however, a discrete-time solution will suit our needs nicely. This involves firing a photon and tweaking its trajectory and velocity at every time step of the simulation (every 17ms, in this case). This calculation can be broken down into a number of steps:

1. Project the photon's velocity ahead by one additional time step
2. Calculate the index of refraction, n_2, at the projected position using Equation 1
3. Find the angle, θ_1, between the photon's velocity vector and its normal vector to the black hole
4. Calculate the new angle, θ_2, using Equation 3 and the previous index of refraction as n_1.
    a. When $n_2 < n_1$, calculate the critical angle of reflection and adjust θ_2 appropriately
5. Rotate the velocity vector by an angle $\theta_2 - \theta_1$
6. Repeat steps 3 – 5 for every black hole
7. Set the velocity's magnitude according to Equation 1
8. Calculate the photon's new position using $x = vt$
9. Repeat steps 1 – 9 for every simulated photon

IV. SIMULATION SPECIFICS

The code for this simulation can be found at [4]. It is programmed in Java 8 and only depends on Apache Commons FastMath for calculations. It is based on real-world units: every pixel on screen is approximately 500 miles, making the photon motion accurate to real-world timing. Buttons on the top left control different modes of the application. Double clicking on empty space will launch a radial blast of photons, right clicking will add/remove a black hole, and using the scroll wheel will change the mass of a black hole. Default mass is $10{,}000 M_{\odot}$.

V. RESULTS

The Java application can be used to simulate a number of real-world and hypothetical situations regarding black holes. One of the simplest is to simulate volleys of parallel photons toward a single black hole (Figure 3). Note the curved path of the photons: farther from the black hole, photons curve to a lesser extent. Closer in, photons will impact or even orbit the black hole. While the simulation does find a 'photon sphere' (the closest stable orbit) at the correct location, no photon orbit around a single black hole is stable, including at the photon sphere. Thus, every photon that orbits the black hole is doomed to degrade. Photons that fall into the black hole experience time dilation and slow to a stop as the index of refraction approaches zero at the event horizon.

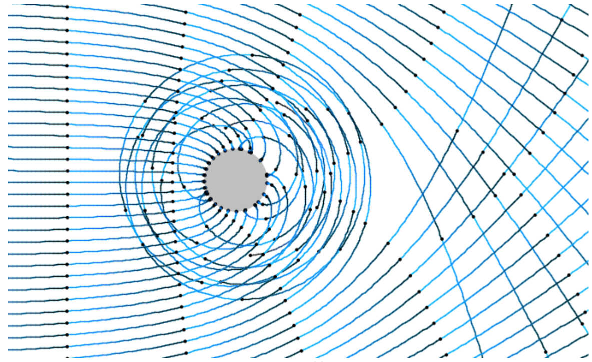

Figure 3 – Parallel incoming photons affected by a single 10,000 solar mass black hole. The grey circle is the event horizon.

Photons can also be simulated near a grouping of black holes (Figure 4). When more than one black hole is affecting the photon, not all orbits are doomed to fall into the event horizon. Instead, photons may find stable orbits or eventually escape into space. Interaction between black holes is not simulated. An interesting phenomenon occurs when multiple black holes are present: because gravitational potential is additive, the event horizon of each black hole grows. Figure 5 demonstrates how, in this model, event horizons between black holes will grow, interact, and even merge. I am unsure of whether this is representative of real-world interaction or simply an artifact of the Fermat Surface approximation.

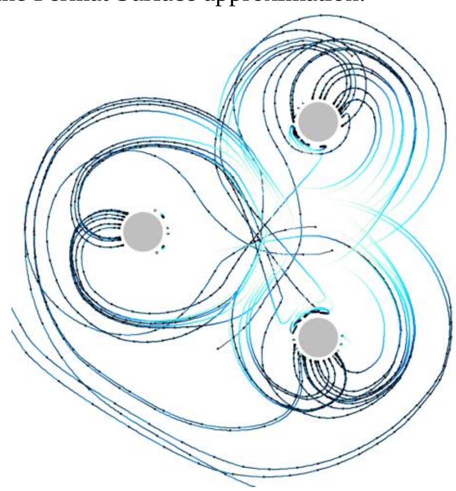

Figure 4 – Photons orbiting three equally-sized black holes.

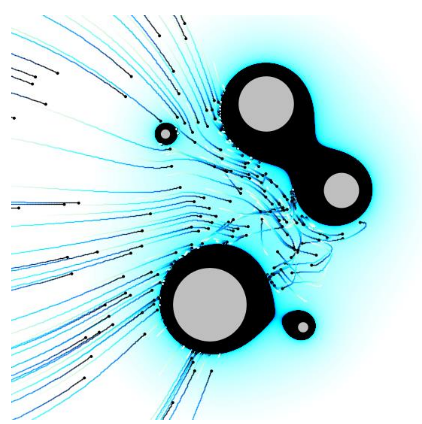

Figure 5 – Photons on top of a heatmap of refractive index. Black regions signify a negative refractive index, AKA the event horizon.

Another application of this simulation can be to visualize the effect of Schwarzschild's general relativistic correction[5] on the gravitational potential of photons, as in the second term of Equation 2. The paths of photons plotted in Figure 6 demonstrate the differences between a solution with purely Newtonian gravitational potential (blue) and using the correction (red). The photons with corrected potential maintain longer and more stable orbits, and are generally more strongly drawn toward the black hole.

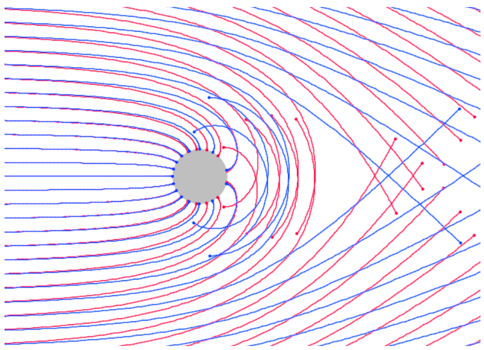

Figure 6 – Photons simulated using only Newtonian approximations for potential (blue) and with the Schwarzschild correction (red)

This simulation may hold merit for testing certain theories regarding general relativity as well. For instance, a paper published by NASA in May 2016[6] proposes that Dark Matter may be composed of microscopic, low mass primordial black holes. A rudimentary simulation sending photons through a field of a hundred $0.1M_{\odot}$ black holes (Figure 7) results in relatively straight paths only with occasional large deflections from light rays coming very close to one of the black holes. This sort of 'scattering' might be observed to test the paper's hypothesis. This simulation is likely not accurate under such extreme conditions, though, and should only serve as insight into the possible uses of a more comprehensive discrete-time photon simulation.

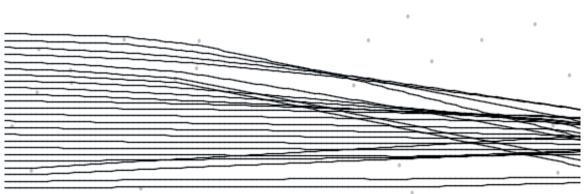

Figure 7 – Deflected photon paths moving left-to-right in a field of small, scattered black holes

IV. CONCLUSION

Discrete time-step simulations of photon motion along the curvature of space-time allow for more flexibility and complexity in the scene when compared to continuous solutions. Fermat's treatment of curvature as a continuous refractive index is a good starting point for this type of simulation. However, possible compounding errors and strange motions can occur in extreme situations, such as when a photon experiences reflection due to exceeding the critical angle at a given time step. While still very useful for visualization, these issues make the simulation program in its current state unsuitable for general relativity research in anything but the most simple of simulations. However, with further work to account for all general relativistic effects, discrete-time photon simulations could be invaluable in advancing our understanding of space-time curvature and general relativity. Interactivity and real-time physics give this approach value in the possible rapid testing and development of new theories in general relativity.

V. ACKNOWLEDGEMENTS

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VI. REFERNCES

[1] https://en.wikipedia.org/wiki/Schwarzschild_geodesics
[2] https://en.wikipedia.org/wiki/Gravitational_lensing_formalism
[3] Massimo Meneghetti, Introduction to Gravitational Lensing, p. 13
[4] https://github.com/tornato7/PhotonTrajectorySim
[5] http://www2.warwick.ac.uk/fac/sci/physics/current/teach/module_home/px436/notes/lecture15.pdf
[6] Kashlinsky, A., 2016b, ApJL, 823, L25
[7] Evans, J., Am. J. Phys. 69, 1103 (2001)
[8] Harwit, A., Astrophys.J.597:1266-1270, 2003